A Traveling-Wave Directional Filter*

FRANKLIN S. COALE†

Summary—**A new type of microwave filter is presented in which resonance occurs in the form of a traveling wave rather than in the conventional form of a standing wave. This device is a constant-resistance circuit, and therefore presents a very low input vswr. Formulas for loaded Q and insertion loss are given. Experimental results verify the theoretical approach. This filter, which is constructed of a transmission-line loop and two directional couplers, finds application in multiplexing filters as well as in matched band-pass and band-rejection filters.**

Introduction

MICROWAVE filters usually employ one or more elements which are resonant in the sense that a high standing wave is present. It is possible, however, to obtain resonance in another sense. Consider a directional coupler in which the coupled and isolated arms are connected to form a ring of electrical length corresponding to n wavelengths. A signal incident at the primary arm of the directional coupler will cause in the secondary, a resonance, that is in the form of a traveling wave circulating about the loop.[1-3] This process of build-up to resonance is similar to standing-wave build-up in a microwave cavity.

If two directional couplers are connected in the loop (as shown in Fig. 1) at the resonant frequency of the loop all the energy will pass from the primary arm of one directional coupler (arm 1) to the primary arm of the other (arm 4). The device will then behave as a band-pass filter with an unloaded Q equal to the Q of the transmission line, and a loaded Q dependent upon the voltage coupling coefficient of the directional couplers. At resonance, the through arm of the directional coupler behaves as a band-rejection filter (arm 1 to arm 2). This type of filter will be referred to as a "traveling-wave directional filter."

The traveling-wave directional filter is a four terminal-pair network with the frequency response shown in Fig. 2, and lends itself quite naturally to multiplexing filter applications. By using traveling-wave directional filters as basic building blocks, it is possible to construct multiplexing filters with a perfect match, since the input vswr of a directional filter is theoretically unity both at resonance and off resonance, and thus avoid the off-resonance mismatch that is probably the most difficult problem associated with the ordinary multiplexing filters (*i.e.*, those constructed with simple band-pass, band-rejection, high-pass, and low-pass filters).

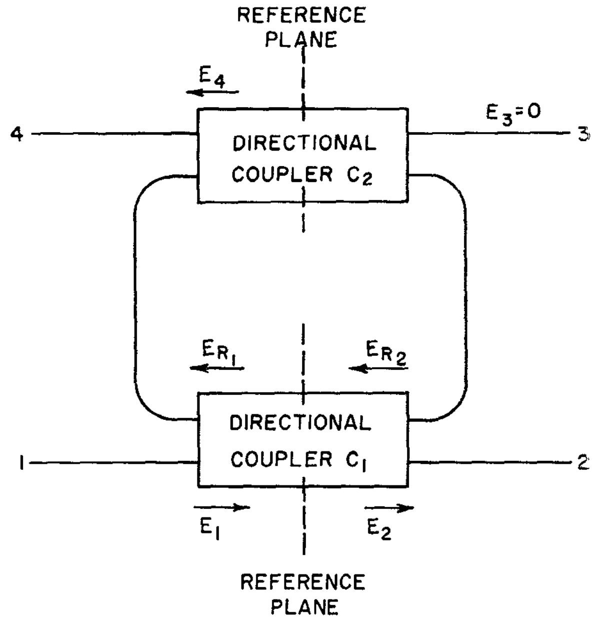

Fig. 1—Schematic of traveling-wave directional filter.

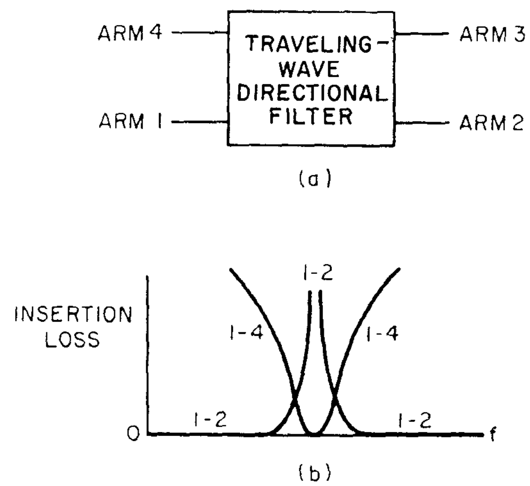

Fig. 2—Response of a traveling-wave directional filter.

Theoretical Analysis

Two different approaches are used to analyze the traveling-wave directional filter. By assuming that all elements of the filter are adjusted so that a pure traveling wave exists in the loop, it is possible to calculate such quantities as loaded Q and insertion loss simply, in terms of the coupling and the length and attenuation constant of the loop transmission line. The second ap-

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† Stanford Res. Inst., Menlo Park, Calif.

[1] B. H. Ring, U. S. Patent 2,639,326, issued May 19, 1953.

[2] P. J. Sferazza, "Traveling wave resonator," *Tele-Tech*, vol. 14, pp. 84–85, 142–143; November, 1955.

[3] K. Tomiyasu, "A new annular waveguide rotary joint," Proc. IRE, vol. 44, pp. 548–553; April, 1956.

proach is somewhat more involved, but is capable of predicting anomalies due to imperfect adjustment of the filter. Because of the presence of the directional couplers, it is possible to have two mutually orthogonal standing-wave resonances in loop, corresponding to electric and magnetic coupling. When the two resonant frequencies are made to coincide, the field components of the two resonant modes superimpose to form a pure traveling wave in the clockwise direction around the loop.

It is assumed that: 1) all transmission lines have the same characteristic impedance, 2) all directional couplers are perfectly matched and have infinite directivity, and 3) no points of reflection exist in the loop and therefore a pure traveling wave exists. The expressions for loaded Q and insertion loss are derived for the TEM-transmission-line case, and the expression for loaded Q must be modified as shown later to be applicable to the waveguide-loop case. The voltages shown in Fig. 1 at the terminals of the directional couplers are, for convenience, referred in amplitude and phase to the midplanes of the directional couplers. The total length of the loop at resonance, including the lengths of the coupling regions, is $l=n\lambda$, where n is an integer. It is seen that the following voltages are excited during the first traversal of the loop by the traveling wave:

$$E_1 = 1\angle 0 \quad (1)$$

$$E_2 = \sqrt{(1-c_1^2)}\angle 0 \quad (2)$$

$$E_3 = 0 \quad (3)$$

$$E_4 = c_1c_2e^{-\alpha l/2}\angle 0 \quad (4)$$

$$E_{R_1} = c_1\angle -90 \quad (5)$$

$$E_{R_2} = c_1\sqrt{(1-c_2^2)}e^{-\alpha l}\angle -90 \quad (6)$$

where c_1 and c_2 are the voltage couplings of the directional couplers, and α is the attenuation constant of the loop transmission line.

For the nth traversal of the loop, the voltages may be written in terms of E_{R_2} as follows:

$$E_2 = \sqrt{(1-c_1^2)} - c_1E_{R_2}\angle 0$$

$$E_3 = 0$$

$$E_4 = \frac{c_2e^{\alpha l/2}}{\sqrt{(1-c_2^2)}}E_{R_2}\angle 0$$

$$E_{R_1} = \frac{e^{\alpha l}}{\sqrt{(1-c_2^2)}}E_{R_2}\angle -90$$

where

$$E_{R_2} = c_1\sqrt{(1-c_2^2)}e^{-\alpha l} \cdot \left\{1+\sum_{m=1}^{m=n}(1-c_1^2)^{m/2}(1-c_2^2)^{m/2}e^{-m\alpha l}\right\}\angle -90.$$

Letting $n\to\infty$ and summing this expression in closed form, we obtain

$$E_{R_2} = \frac{c_1\sqrt{(1-c_2^2)}e^{-\alpha l}}{1-\sqrt{(1-c_1^2)}\sqrt{(1-c_2^2)}e^{-\alpha l}}\angle -90. \quad (7)$$

The condition for perfect rejection between arms 1 and 2 may be obtained by setting $E_2=0$, which yields

$$e^{2\alpha l} = \frac{1-c_2^2}{1-c_1^2}. \quad (8)$$

For perfect rejection, (8) requires that $c_2<c_1$ if $\alpha>0$. With $c_2=0$, the case of a single line coupled to a loop is obtained. The condition for maximum output from E_4 for a given bandwidth can be shown to be $c_1=c_2=c$. Thus, when attenuation of the loop is considered, the conditions are different for obtaining $E_2=0$ and $E_4=$ maximum.

The loaded Q of the filter may be calculated assuming $c_1=c_2=c$. At resonance the output voltage is

$$E_4 = \frac{c^2e^{-\alpha l/2}}{1-(1-c^2)e^{-\alpha l}}. \quad (9)$$

At a different frequency which corresponds to an electrical length around the loop equal to $2\pi n+\theta$, the output voltage is

$$E_4 = \frac{c^2e^{-1/2(\alpha l+j\theta)}}{1-(1-c^2)e^{-(\alpha l+j\theta)}}. \quad (10)$$

Defining $Q_L=f_o/2(f_o-f_1)=n\pi/\theta_1$, where f_1 and θ_1 are values at the half-power point, we have from (9) and (10),

$$E_4 = \left|\frac{c^2e^{-1/2(\alpha l+j\theta_1)}}{1-(1-c^2)e^{-(\alpha l+j\theta_1)}}\right| = \frac{\sqrt{2}}{2}\frac{c^2e^{-\alpha l/2}}{1-(1-c^2)e^{-\alpha l}}.$$

Taking absolute magnitudes of each side and simplifying, we may solve for θ_1 in terms of c and α:

$$\cos\theta_1 = 1-\frac{[1-(1-c^2)e^{-\alpha l}]^2}{2(1-c^2)e^{-\alpha l}}. \quad (11)$$

If θ_1 is small, we may replace $\cos\theta_1$ by $1-(\theta_1^2/2)$ which yields

$$\theta_1 = \frac{1-(1-c^2)e^{-\alpha l}}{\sqrt{(1-c^2)}e^{-\alpha l/2}}. \quad (12)$$

The loaded Q of this filter[4] is

$$Q_L = \frac{n\pi\sqrt{(1-c^2)}e^{-\alpha l/2}}{1-(1-c^2)e^{-\alpha l}}. \quad (13)$$

It should be noted that this expression gives the true loaded Q of the resonator as it would be measured, and

[4] When a waveguide loop is used in the filter, Q_L as given in (13) must be multiplied by $(\lambda g/\lambda)^2$ where λg is the guide wavelength and λ is the free-space wavelength. Eq. (14) for insertion loss applies without change to the waveguide case.

takes account of the external loading of the couplers combined with the losses in the transmission of the loop.

The insertion loss at resonance is given by 20 log E_1/E_4, and, since $E_1=1$, we have

$$\text{Insertion Loss} = 20 \log \frac{1-(1-c^2)e^{-\alpha l}}{c^2 e^{-\alpha l/2}}. \qquad (14)$$

Eqs. (13) and (14) were derived assuming equal characteristic impedances of the transmission lines and assuming c to be the voltage coupling, but they apply correctly to unequal impedance of the ring and input and output arms if c^2 is defined as the power coupling of the directional couplers. For small couplings, c^2 may be expressed as a function of Q_2 and insertion loss as

$$c^2 = \frac{n\pi}{Q_L 10^{I.L./20}}. \qquad (15)$$

If the insertion loss is small we have

$$c^2 = \frac{n\pi}{Q_L}. \qquad (16)$$

The foregoing analysis is applicable only when the filter is perfectly tuned. It is possible, however, to analyze the problem by assuming two resonant orthogonal modes corresponding to electric and magnetic coupling from the external transmission lines to the loop. The discussion will now be confined to shielded-strip-line directional couplers as developed at Stanford Research Institute.[5,6] (The principle applies equally well for all directional couplers.) The analysis of footnotes 5 and 6 utilizes the two basic TEM modes that may exist on the pair of coupled strips (*i.e.*, the even and odd modes). Fig. 3 shows the electric fields of these two modes, in which the strips are either at the same potential or at opposite potentials (corresponding to even and odd modes, respectively). The impedance of the strip line is dependent upon the polarity of the strips in the coupling region. It has been shown that $Z_o' = \sqrt{Z_{oo}Z_{oe}}$ where Z_o' is the characteristic impedance of a single strip line that will match the directional coupler; Z_{oo} and Z_{oe} are the odd- and even-mode characteristic impedances of the coupled strips, respectively. The odd mode corresponds to electric coupling between the strips, and the even mode to magnetic coupling. Because of the differences in impedance of the resonant loop, it may be treated as a length of nonuniform transmission line as shown in Fig. 4. In general, the resonant frequency of the even-mode transmission line will differ from that of the odd-mode line. It must be noted that, in this analysis, the directional coupler is considered to be a device which couples energy (nondirectionally)

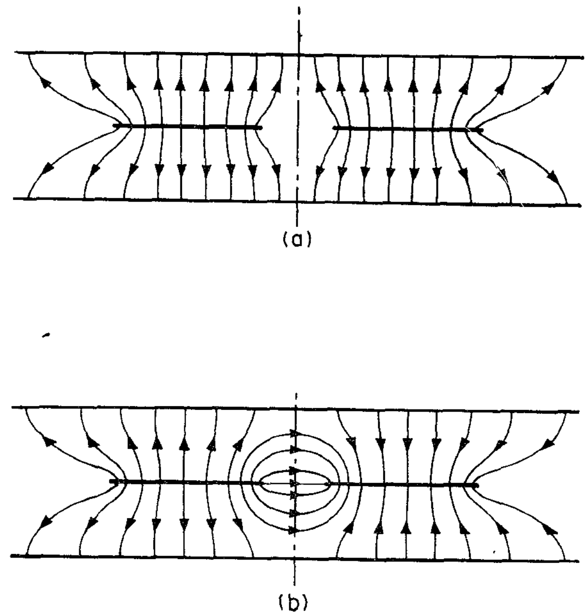

Fig. 3—Odd and even mode electric field distributions existing in a pair of coupled strips. (a) Even-mode electric field distribution. (b) Odd-mode electric field distribution.

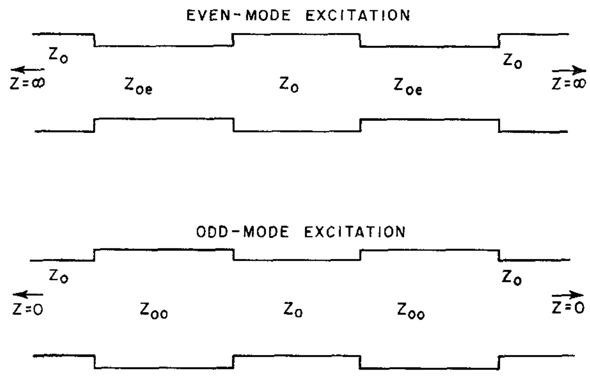

Fig. 4—Nonuniformity of even and odd mode traveling-wave loop transmission lines.

to the loop in the form of two orthogonal resonant modes. At a frequency for which the electric (odd) mode is resonant, waves travel in opposite directions in the loop, forming a standing wave with voltage maxima at the centers of the coupling regions and voltage minima at the sides of the loop. Fig. 5 illustrates this case and also the magnetic coupling case for which resonance may occur at a slightly different frequency. Each resonant mode alone yields no directivity, but if the modes are made to resonate at the same frequency—either by a judicious choice of the strip lengths and widths in the loop or by reducing the higher resonant frequency by means of tuning screws at the voltage maxima—the two modes superimpose into a pure traveling wave. Under these conditions the filter obeys the theory derived in the previous section.

The difference in resonant frequency of the two modes may be made equal to zero by choosing the characteristic impedance of the sides of the loop to be $Z_o = \sqrt{Z_{oe}Z_{oo}}$. However, tuning devices are usually still

[5] S. B. Cohn, "Shielded coupled-strip transmission lines," IRE Trans., vol. MTT-3, pp. 29-38; October, 1955.

[6] E. M. T. Jones and J. Bolljahn, "Coupled strip line filters and directional couplers," IRE Trans., vol. MTT-4, pp. 75-81: April, 1956.

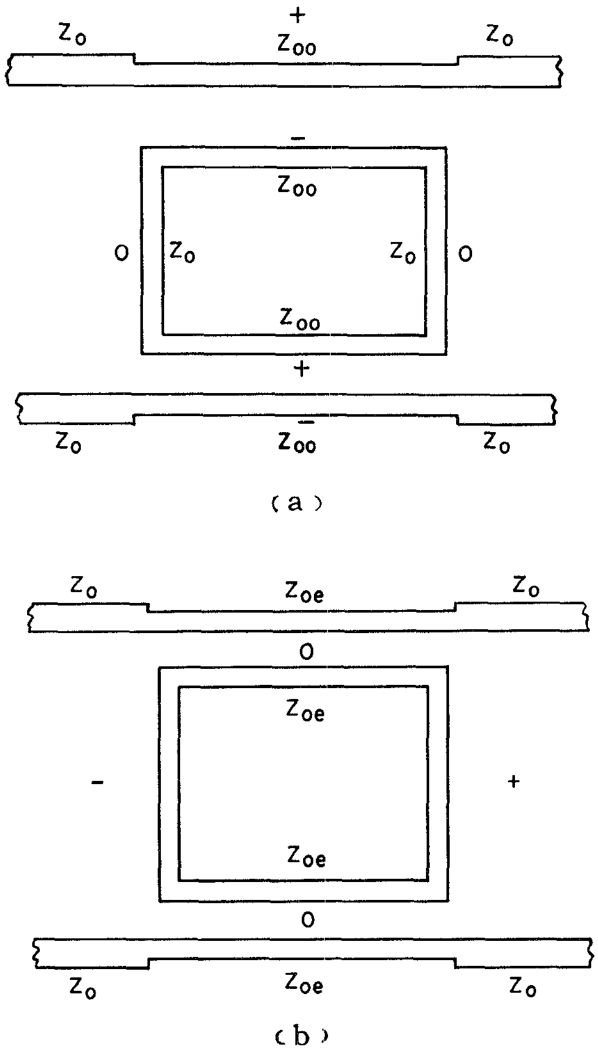

Fig. 5—(a) Electric field coupling to loop.
(b) Magnetic field coupling to loop.

necessary to compensate for points of reflection that might otherwise destroy the orthogonality of the two modes. It can be shown that three shunt capacitive susceptances, spaced at quarter-wavelength intervals around the loop, will serve both to tune the two modes to the desired frequency, and to eliminate any spurious coupling between the modes. However, in practice, four susceptances are somewhat easier to tune and offer less deterioration of the insertion loss.

Design and Experimental Results

A typical design procedure for a traveling-wave directional filter might be as follows.

The filter specifications should include such items as resonant frequency, loaded Q (or bandwidth), maximum allowable insertion loss in the pass band, and the characteristic impedance of both the input and output. It is first necessary to select tentatively a strip-line cross section and perform a preliminary theoretical check to ascertain whether the desired loaded Q and an acceptable pass-band loss may be obtained with the attenuation constant[7] afforded by this cross section.

From (13) and (8), it is possible to calculate the values of c_1 and c_2, which will be equal in the case of minimum pass-band loss and unequal in the case of perfect rejection. Eqs. (13) and (14) are generally accurate for the case of low attentuation when $c_1 \neq c_2$ if one uses the average value of c_1 and c_2 for c. The formulas in the article on directional couplers by Jones and Bolljahn[6] may be employed to calculate, from c_1 and c_2, the characteristic impedances Z_o, Z_{oo}, and Z_{oe}. From Cohn's article on coupled strip line,[5] the width, w, of each strip and the spacing, s, between the strips may be found from a knowledge of Z_o, Z_{oo}, and Z_{oe}. It is probably best to make the length of the coupling region one-quarter wavelength because the separation, s, is then a maximum for a given coupling value, and because the more nearly square shape of the loop allows the resonant modes to approach a common frequency.

As shown in Fig. 7, the corners of the loop are mitered. Square and round corners were also used, but resulted in a wide frequency separation between the two orthogonal resonant modes.

Since it is impossible to design a loop which will resonate perfectly at a given frequency, it is necessary to provide some means of tuning not only the orthogonal modes but any reflections that might exist in the loop. A simple but effective means of tuning may be achieved by means of four pairs of screws placed at one-quarter wavelength spacings around the loop, preferably at the voltage maxima of the two orthogonal modes.

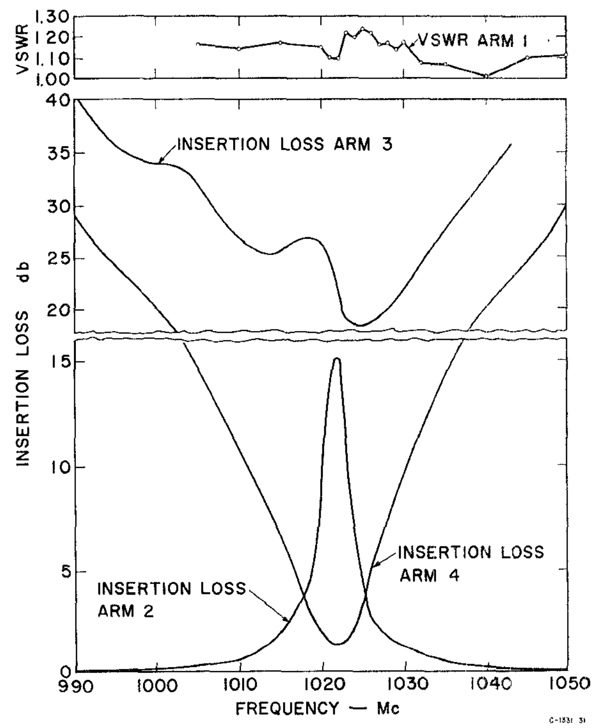

Fig. 6—Frequency response of traveling-wave directional filter.

[7] S. B. Cohn, "Problems in strip transmission lines," IRE Trans., vol. MTT-3, pp. 119–126; March, 1955.

The screws are so located that they may be screwed in evenly from opposite ground planes towards the center of the strip; this affords a symmetrical tuning which will not excite the TEM mode between the parallel ground planes. It should be noted that the screws may be used only to reduce the resonant frequency of the loop. Copper tuning screws allow tuning from 1080 to 1020 mc with a change in insertion loss from 1.1 to 1.3 db.

Fig. 6 gives experimental results for a traveling-wave directional filter. A more complex filter circuit (Fig. 7), containing two separate resonant loops, was also constructed and tested. This circuit separates two frequencies in the 1000 mc region from a spectrum of frequencies incident at the input arm. The experimental loaded Q's of this filter agreed with the theoretical value within 5 per cent. The directional couplers were designed the filter and the thermal properties of the dielectric employed.

Fig. 8 shows a diagram of a double loop filter which has been designed and tested. The typical double-tuned-circuit response is obtained both for the band-pass and band-rejection cases. The coupling coefficient between two identical loops for small coupling values and narrow bandwidths is $K=c/n\pi$, where c is the coupling coefficient of the directional coupler between the two loops and n is the length of the loop in wavelengths. The exact coupling coefficient for two coupled loops is given by

$$K = \frac{1}{n\pi} \tan^{-1} \frac{c}{\sqrt{1-c^2}}. \tag{17}$$

The output voltage for such a filter is given as

$$E_4 = \frac{c_1 c_2 c_3 e^{-1/2(\alpha+j\beta)l}}{(1-\bar{c}_1\bar{c}_2 e^{-(\alpha+j\beta)l})(1-\bar{c}_2\bar{c}_3 e^{-(\alpha+j\beta)l})(1-\bar{c}_1 c_2^2 \bar{c}_3 e^{-2(\alpha+j\beta)l})} \tag{18}$$

to have 16-db coupling at the low-frequency loop and 17-db coupling at the high-frequency loop. One-half-inch plate spacing with polystyrene dielectric was used in this design. The over-all dimensions of this dual filter were 9 by 4.5 by 1 inches. Fig. 7 gives the measured

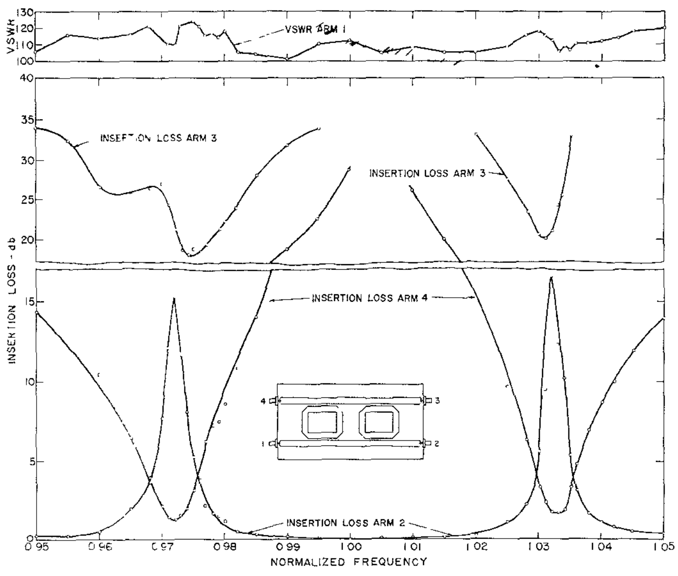

Fig. 7—Frequency response of dual directional filter.

response of this dual-loop filter. A high-power test with 3 kw of CW power at the resonant frequency of the higher loop showed no sign of voltage breakdown. The maximum average power that may be tolerated over a long period of time is limited by the dissipation loss of

where c_1, c_2, and c_3 are the voltage coupling coefficients of the directional couplers, $\bar{c}_1=\sqrt{1-c_1^2}$ etc.

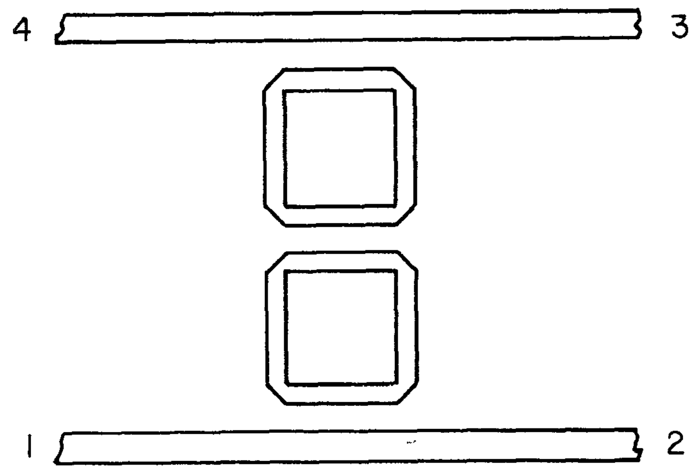

Fig. 8—Layout of double-tuned traveling-wave directional filter.

Conclusion

It may be seen that this type of filter and related types of filters present to the microwave engineer a new type of circuit which is simple in construction, small in size, light weight, and has excellent electrical performance.

Acknowledgment

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